Topological pumping in metamaterials based on symmetry

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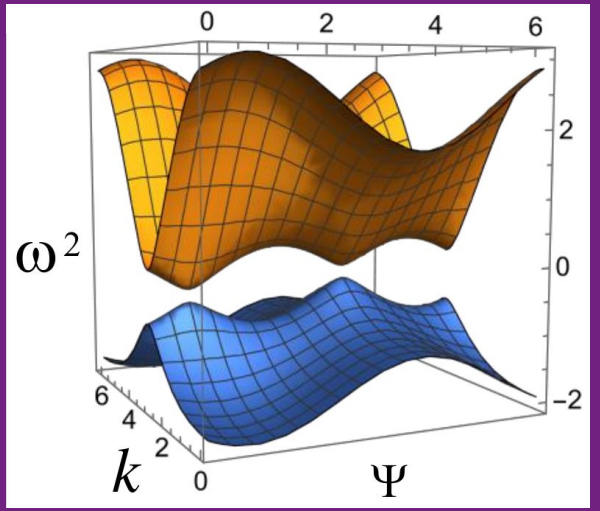
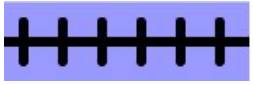
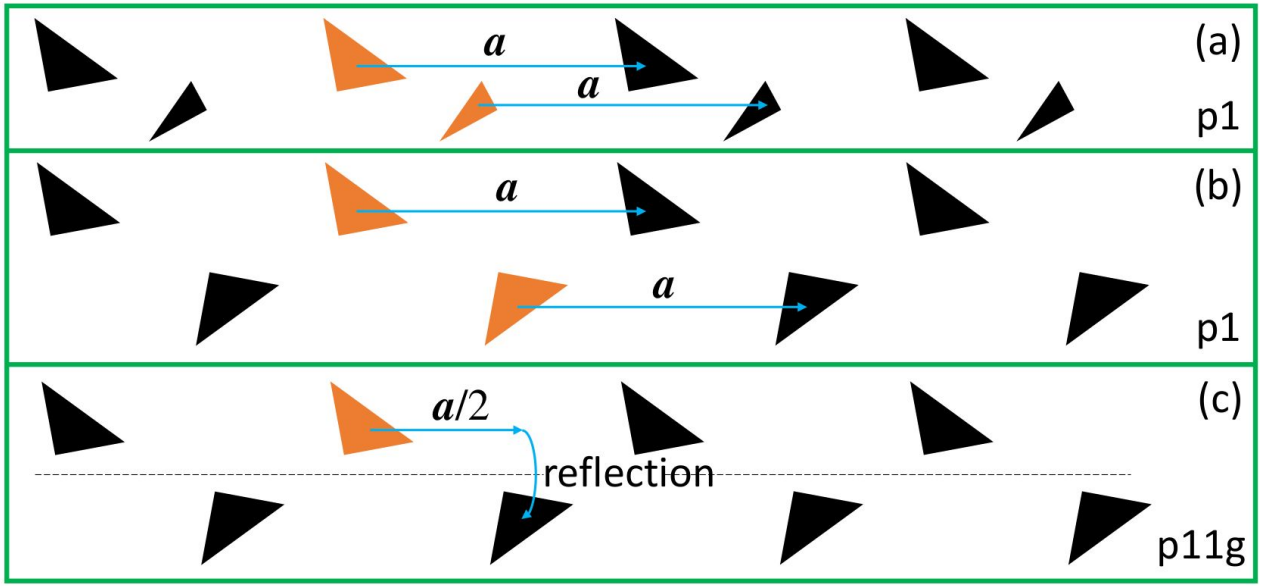


7 frieze groups (1D - periodic chains)

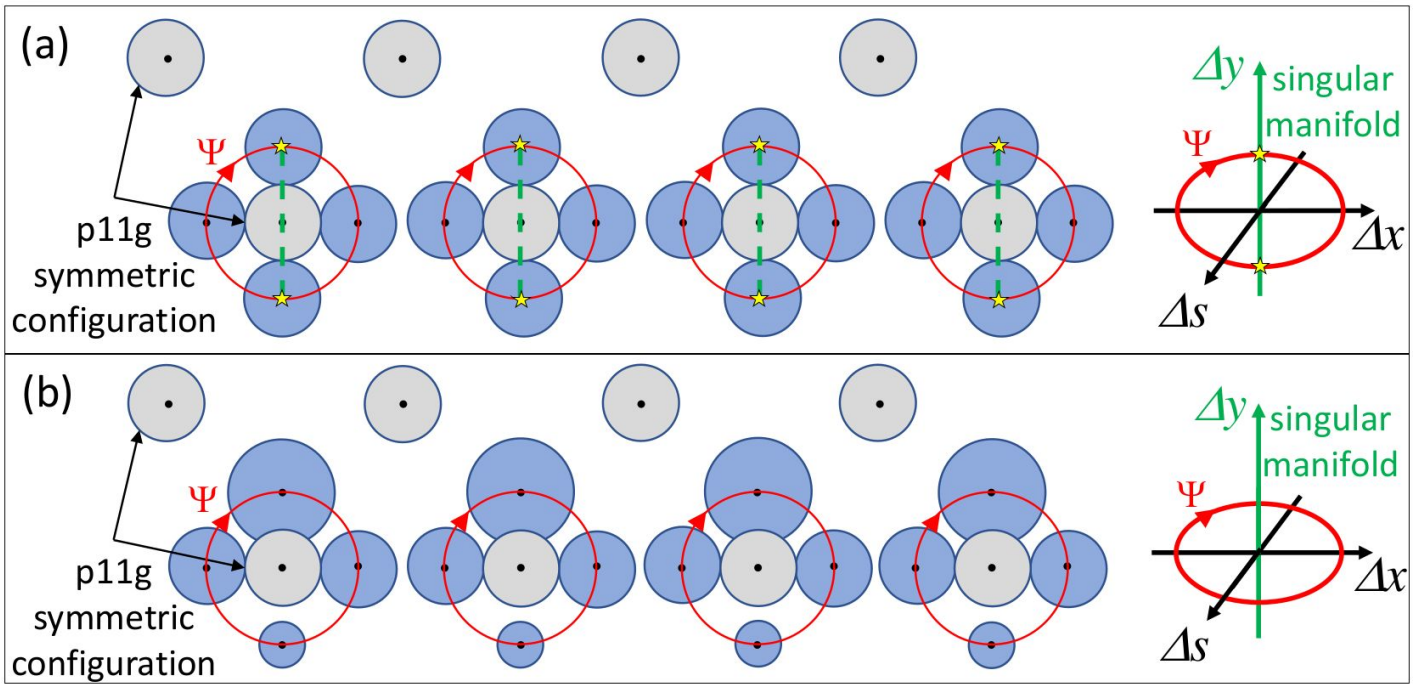
p1			only translations
p11g			glide-reflection
p1m1			vertical reflection
p2			inversion
p2mg			VR + GR
p11m			horizontal reflection
p2mm			VR + HR

From the *p1* group to the *p11g* group



Glide-reflection symmetry reduces the lattice constant by a factor 2 (band folding), resulting in a gap closing

p11g as an isolated manifold in parameter space: scaling



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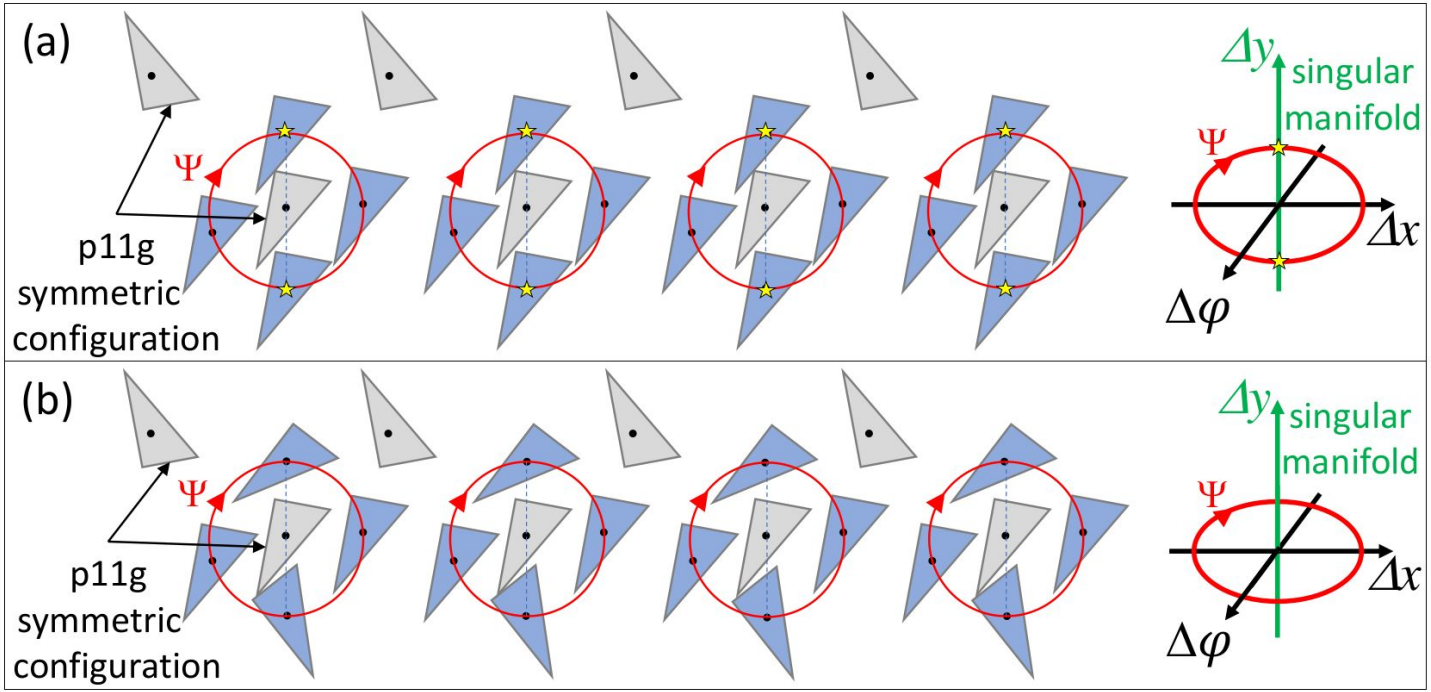
p11g as an isolated manifold in parameter space: rotation



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Claim: topologically non-trivial cycles automatically translate into Thouless pumpings provided the system displays a gap in its resonant spectrum

- A cycle in parameter space is a smooth function of angle Ψ
- The Bloch wavenumber *k* also lives in a circle
- Hence (Ψ,*k*) live on the 2-torus and a Chern number can be defined
- Glide-reflection symmetry provides a gap closing at the X point of the first Brillouin zone

These arguments do not explicitly request a tight-binding model

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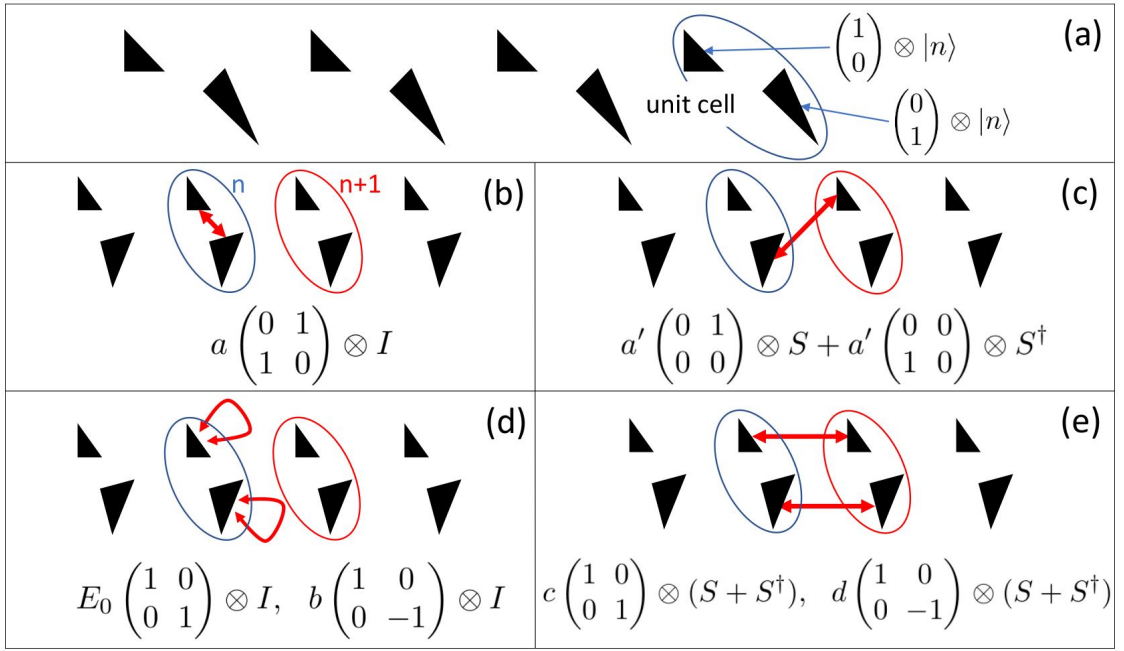
Tight-binding model: 2 energy bands

$$E_{\pm}(k) = E_0 + c\cos(k) \pm \sqrt{(b + d\cos(k))^2 + |a + a' e^{\imath k}|^2}$$

The gap closes for *p11g*:

$$a = a' \text{ and } b = d = 0$$



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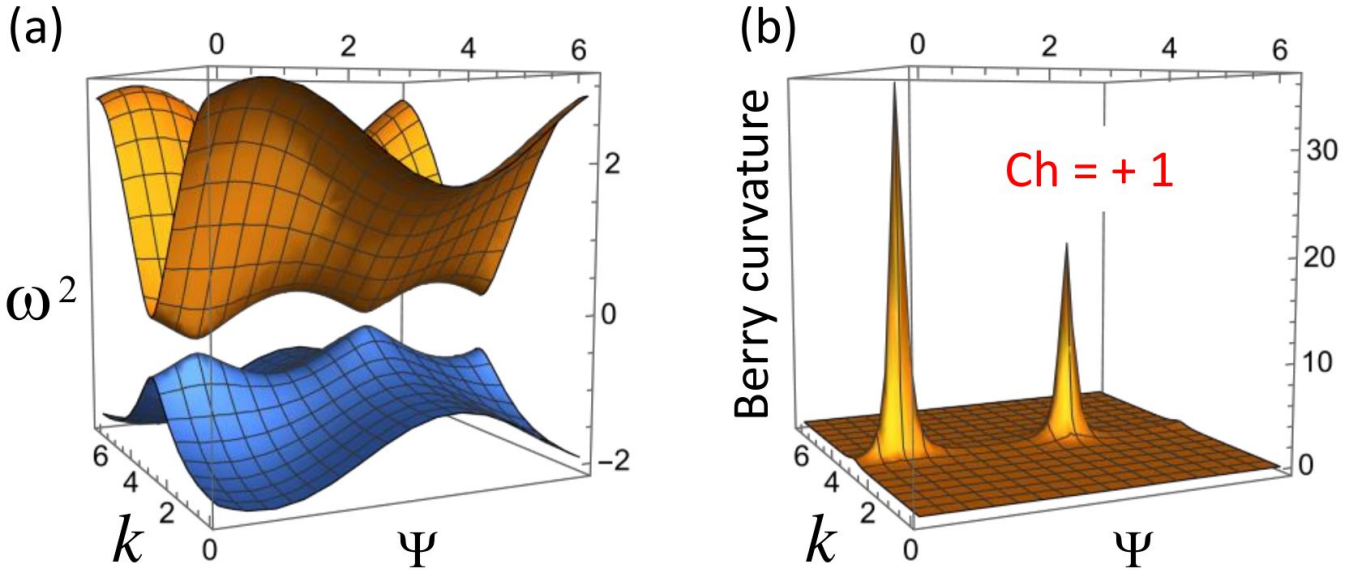
Loop in parameter space

$$a = 1.0 + 0.4\cos(\Psi) \qquad c = 0.4 + 0.1\cos(\Psi - \pi/4)$$
$$a' = 1.0 + 0.4\sin(\Psi) \qquad d = -0.1\cos(\Psi - \pi/4)$$
$$b = 0$$



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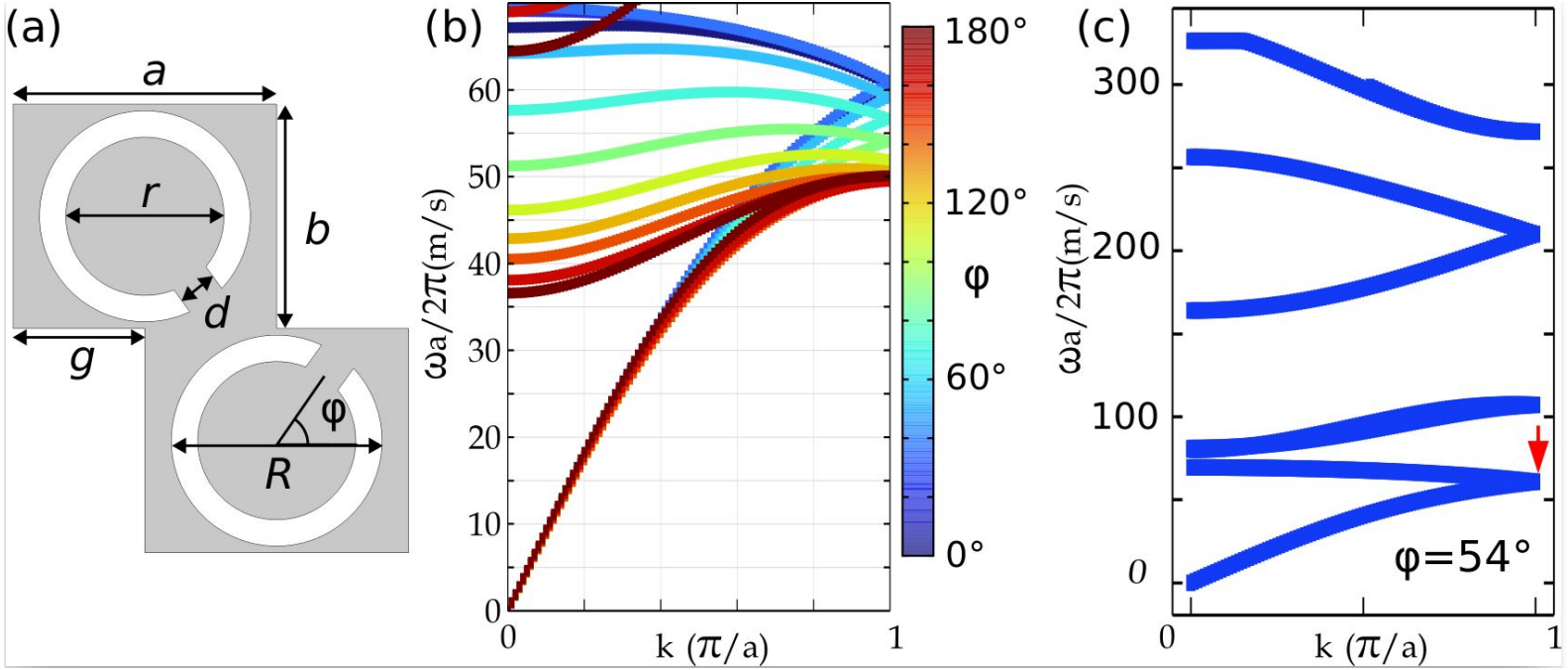
p11g chain of C-shaped acoustic resonators



$b = 0.85a,\ d = 0.15a,\ r = 0.3a,$ and $R = 0.4a$

Hard-wall boundary conditions to define a waveguide

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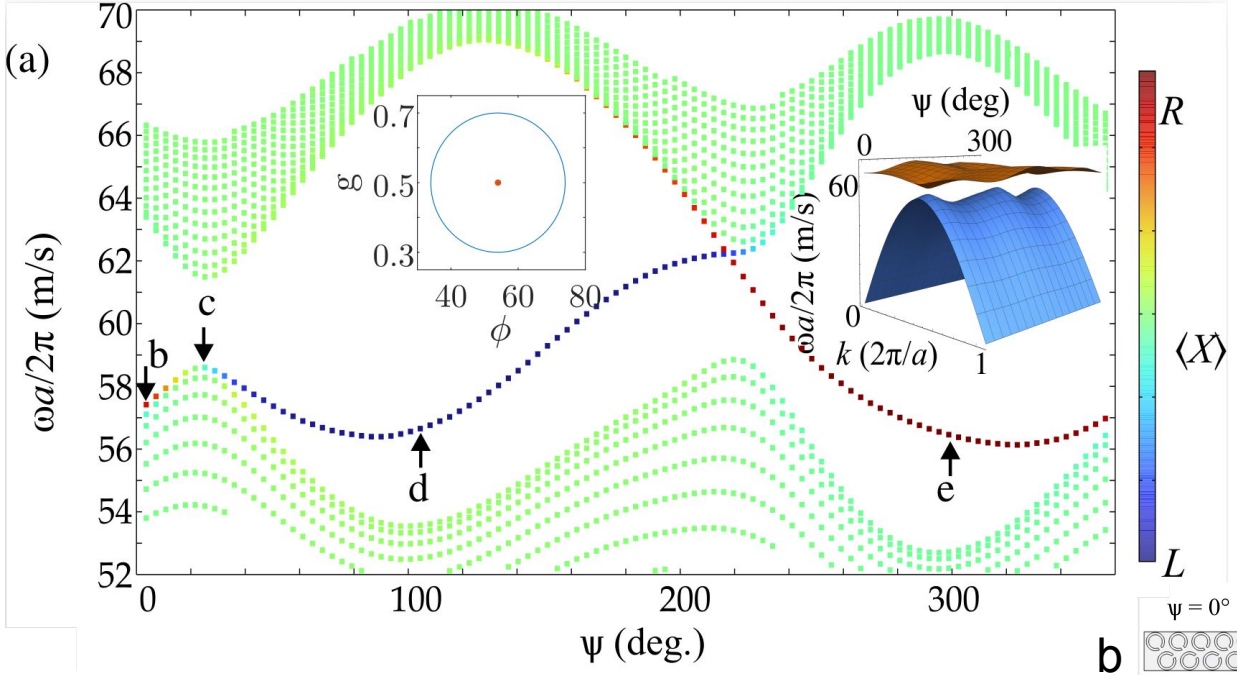
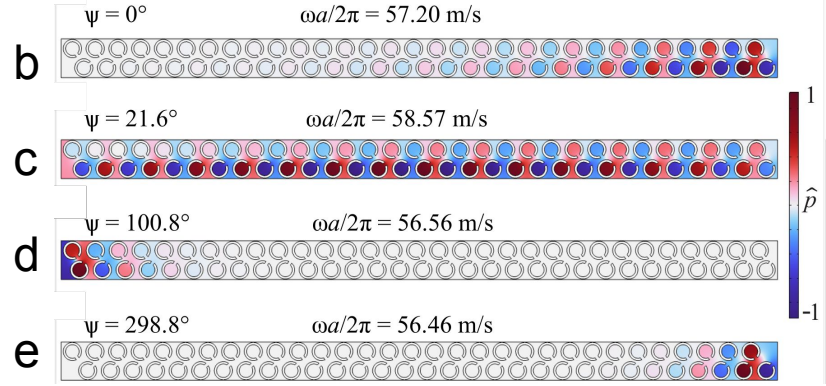
Thouless pumping for a cycle around *p11g*



The cycle is defined purely geometrically



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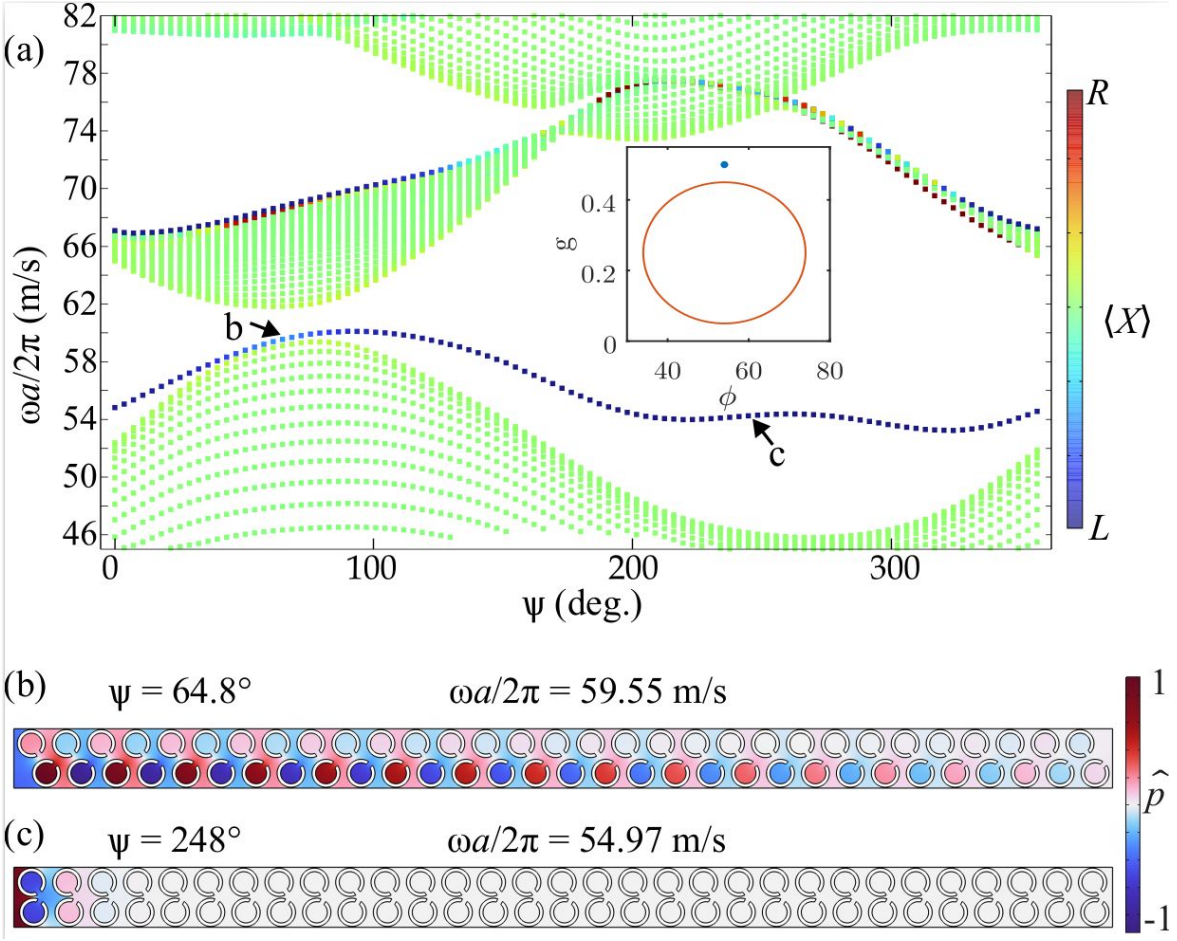
Cycle that does not enclose *p11g*



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Conclusions

- We can identify adiabatic cycles suitable for Thouless pumping, without making appeal to any analytic tight-binding model
- The topological cycles considered were produced using only geometric considerations
- Metamaterial example: simple topological adiabatic cycle for a chain of C-shaped resonators
- Maybe these concepts obtained for frieze groups extend to wallpaper groups and crystallographic groups in 3-dimensions

Julio A. Iglesias Martinez, Muamer Kadic, Vincent Laude, **Emil Prodan**, *Pumping with symmetry*, EPL 146 (2024) 16004

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