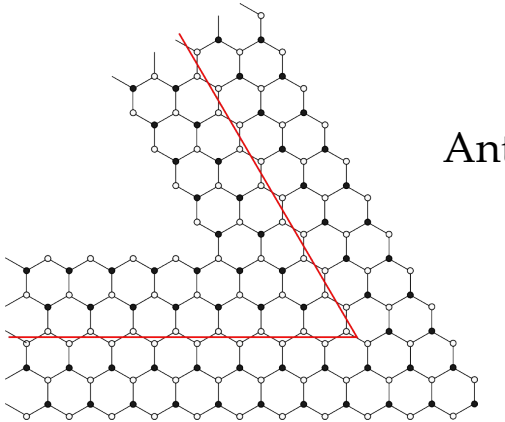
Canonical scattering problem in topological metamaterials

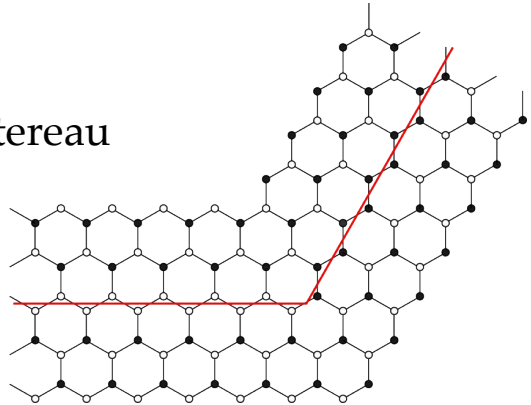
Valley Hall modes through bends

Théo Torres

12 Juin 2024

Antonin Coutant - Cédric Bellis - Régis Cottereau





Laboratoire de Mécanique et d'Acoustique

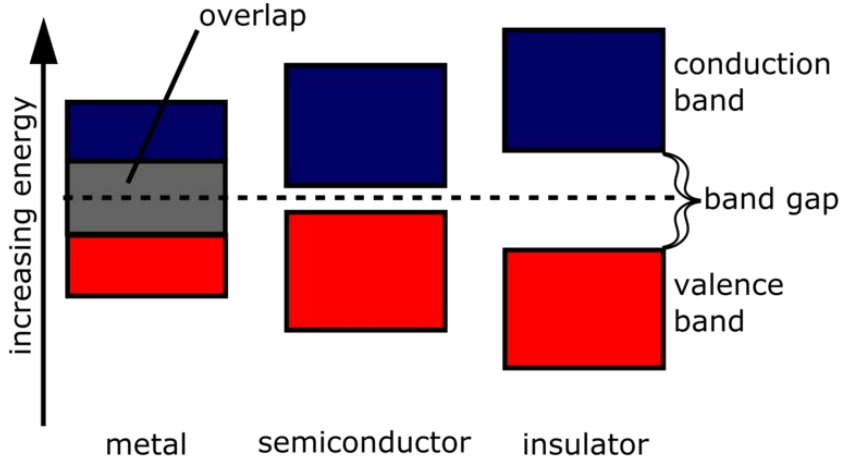
Outline

- Motivation : topological protection and backscattering
- Model and methods
  - Lattice models
  - Scattering matrix formalism
- Analyses of canonical scattering scenarios

Motivation

What are topological insulators ?

Wave propagation in structured media (crystal like structure) → bands and gaps



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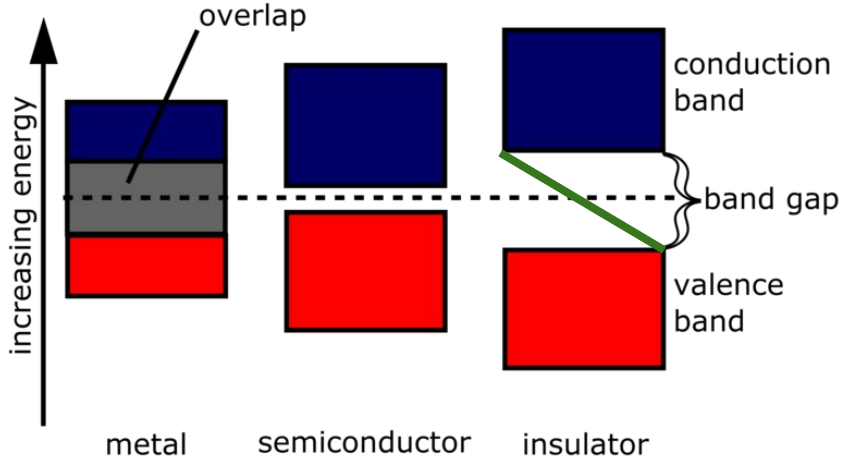


Topological insulators : edge modes

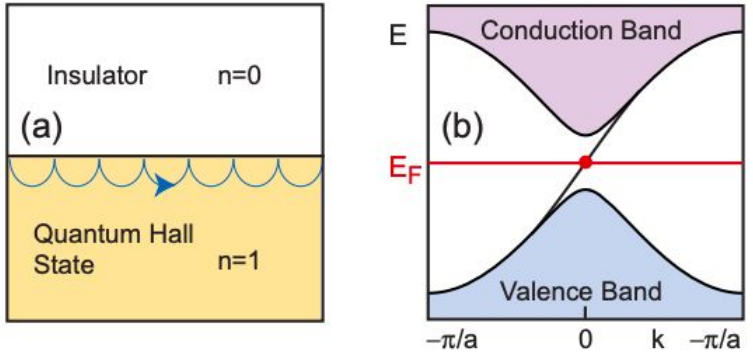
Topological invariant : # edge modes

Interest : “Topological protection”. Smooth transformation will not affect the edge modes.

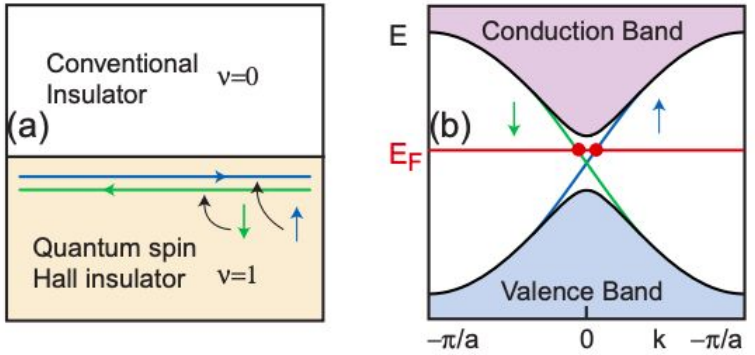
Motivation

Prime examples

The quantum Hall effect
(Break time-reversal symmetry)



The quantum spin Hall effect
(Time-reversal symmetry restored)



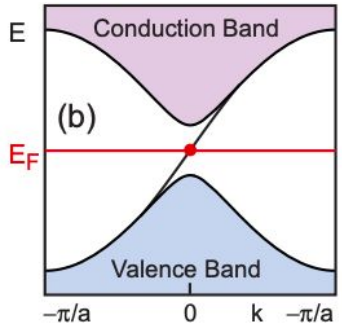
Motivation

- **Topological insulator** : Exotic material characterised by fundamental symmetries that lead to topological invariants.

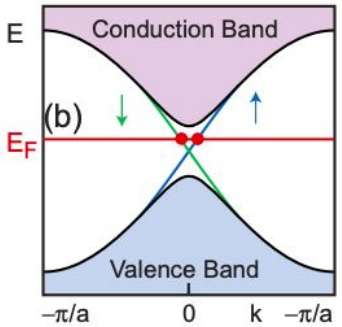
- Topological invariants inform us about the number of edge modes.

- The symmetries have consequences on the scattering (immunity).

QHE :



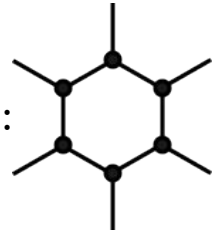
QSHE :



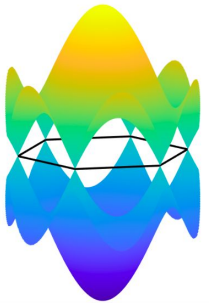
What happens for classical waves ?

Motivation

Analogue Quantum Spin Hall Effect : Create a pseudo-spin.

Honeycomb structure : 

With 2 inequivalent Dirac points



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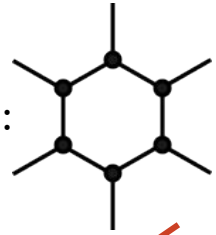
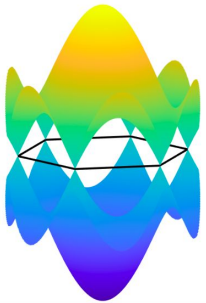
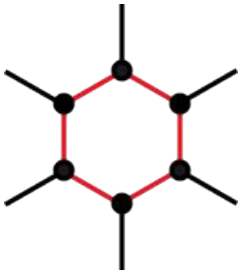
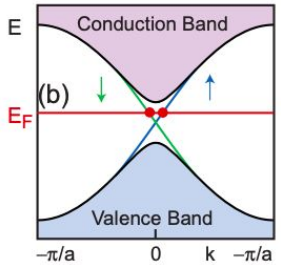
Honeycomb structure : 

With 2 inequivalent Dirac points 

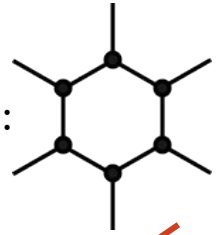


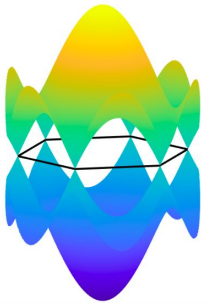


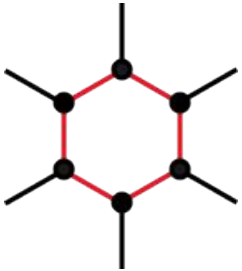
Eg : Kekule - conserved spatial symmetries

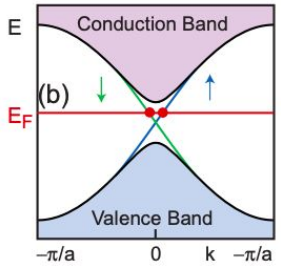
Motivation

Analogue Quantum Spin Hall Effect : Create a pseudo-spin.

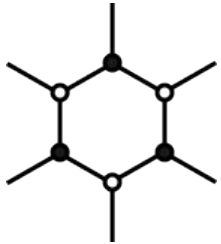
Honeycomb structure : 

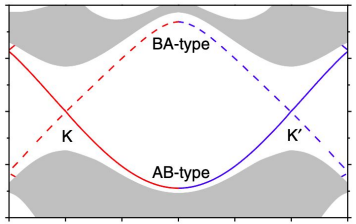
With 2 inequivalent Dirac points 





Eg : Kekule - conserved spatial symmetries



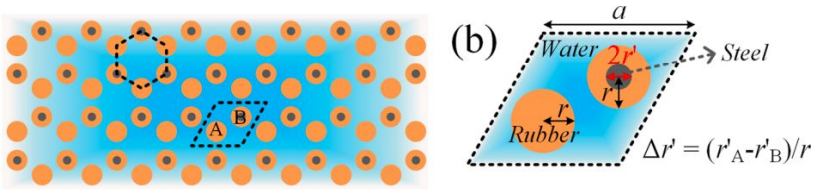


From *Nat Commun* 8, 1304 (2017)

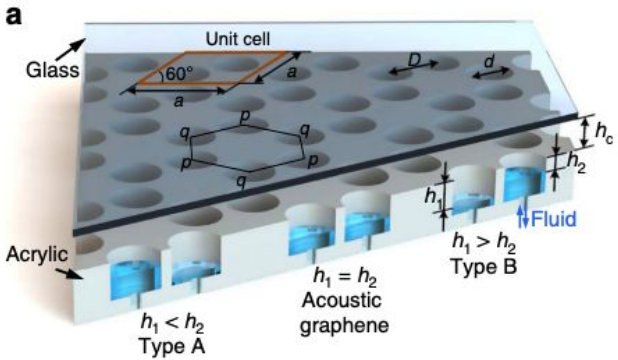
Eg : Valley Hall - broken inversion symmetry

Realisation of the Valley Hall effect

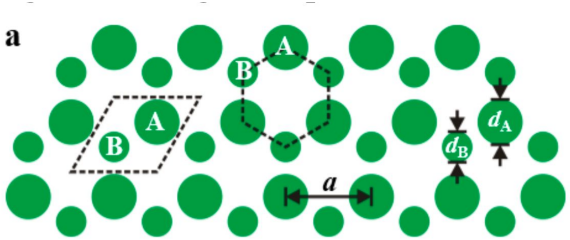
Different realisation



From *Scientific Reports 8, 16784 (2018).*



From *Nat Commun 11, 762 (2020).*

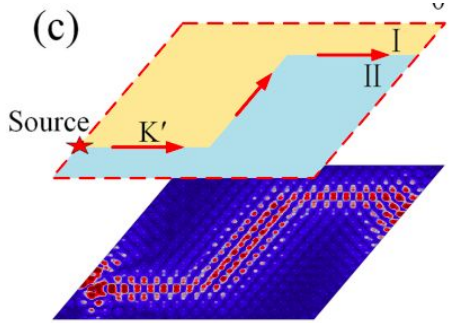


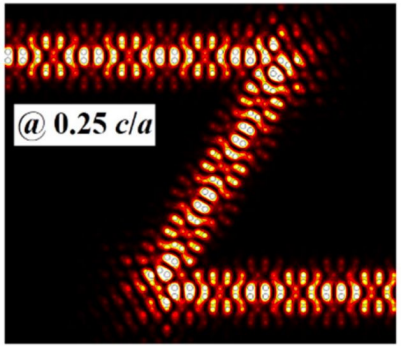
From *Phys. Rev. B* ***96****, 020202(R)*

Realisation of the Valley Hall effect

Testing topological protection

From *Scientific Reports 8, 16784 (2018).*





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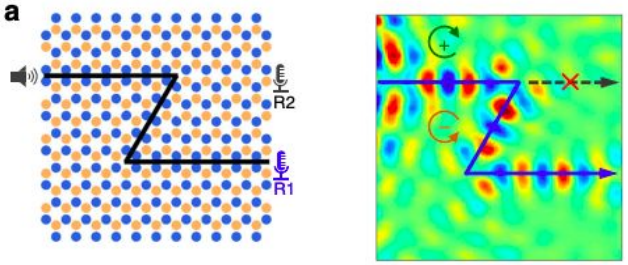


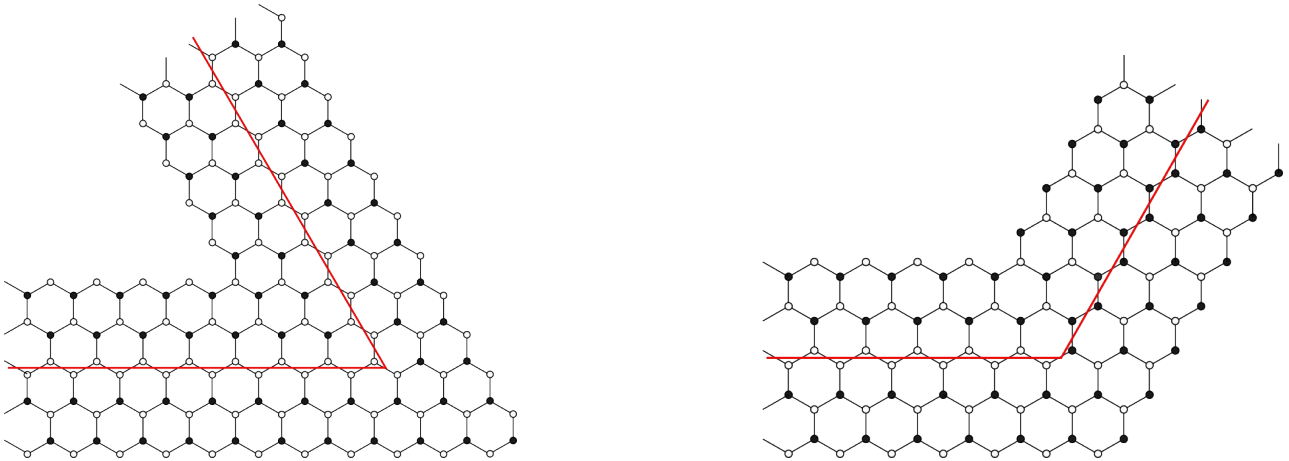
From *Nat Commun 11, 762 (2020).*

Can we quantify the transmission properties ?

What is the impact of the various parameters ?

Model & Methods

Lattice model of graphene :

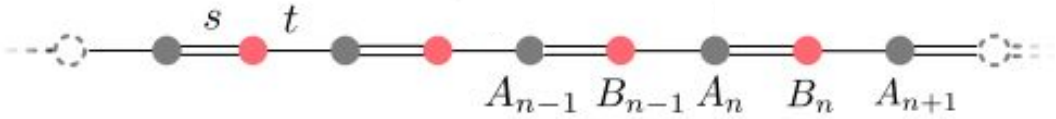


- Lattice models appear as asymptotic models for physical systems
- They allow for a precise definition and analytical expressions for the scattering coefficients

Model & Methods

Transfer matrix formalism :

An example



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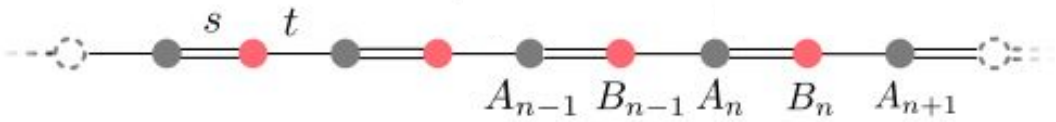

Equations of motion

$$\begin{cases} EA_n = sB_n + tB_{n-1} \\ EB_{n-1} = sA_{n-1} + tA_n \end{cases}$$

Transfer matrix equation

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = M \begin{pmatrix} A_{n-1} \\ B_{n-1} \end{pmatrix} \qquad M = \begin{pmatrix} -\frac{s}{t} & \frac{E}{t} \\ -\frac{E}{t} & \frac{E^2 - t^2}{st} \end{pmatrix}$$

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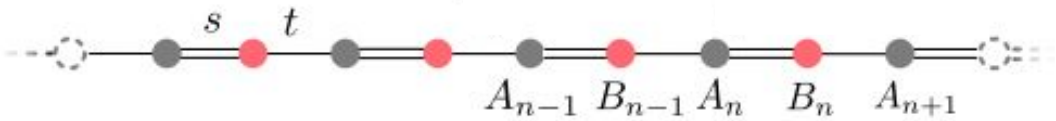

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Mode propagation :

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = e^{ikn} \begin{pmatrix} A \\ B \end{pmatrix} \rightarrow e^{ik} \begin{pmatrix} A \\ B \end{pmatrix} = M \begin{pmatrix} A \\ B \end{pmatrix}$$

Bloch theorem

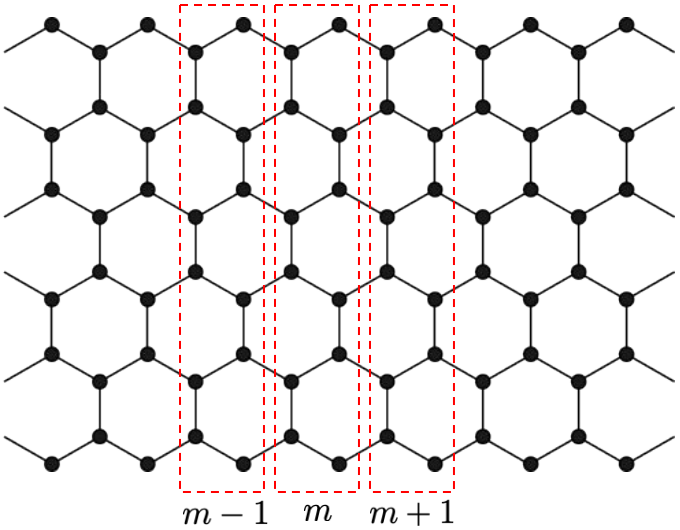
eigenvalues of M

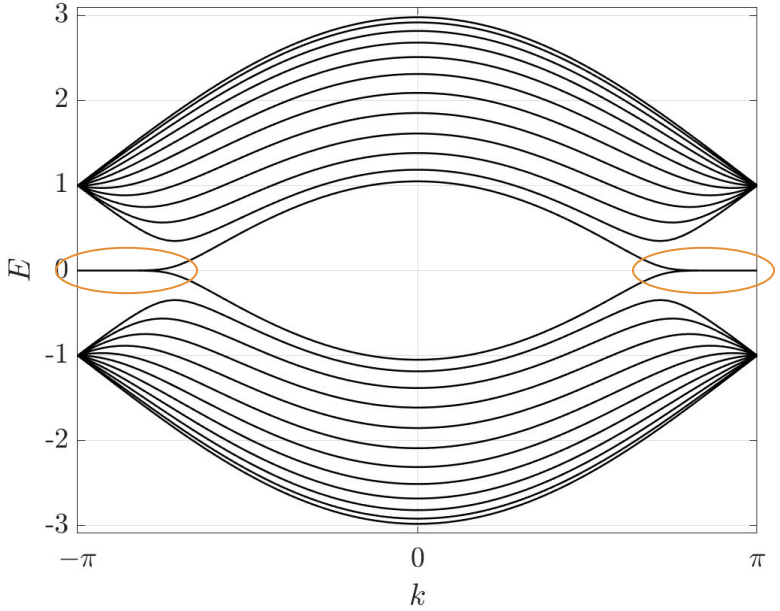
For a fixed E : $\{\lambda_j\}$ spectrum

$$\begin{cases} \text{if } |\lambda_j| = 1 \rightarrow \text{propagating} \\ \text{if } |\lambda_j| \neq 1 \rightarrow \text{evanescent} \end{cases}$$

Analyses - Dispersion relation

$$\phi_m = e^{ikm}\phi$$





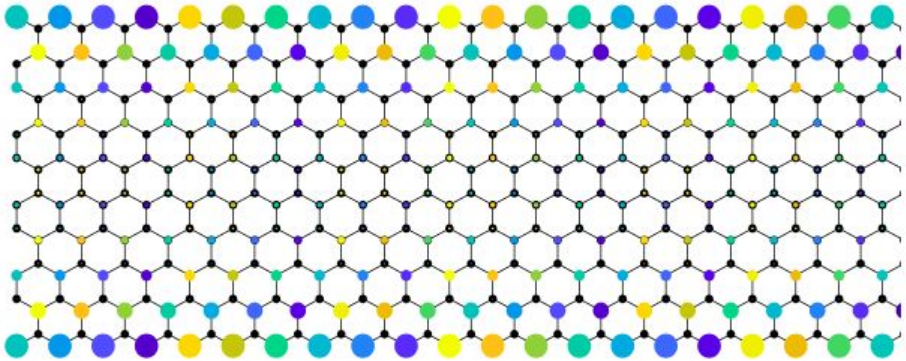
Edge state

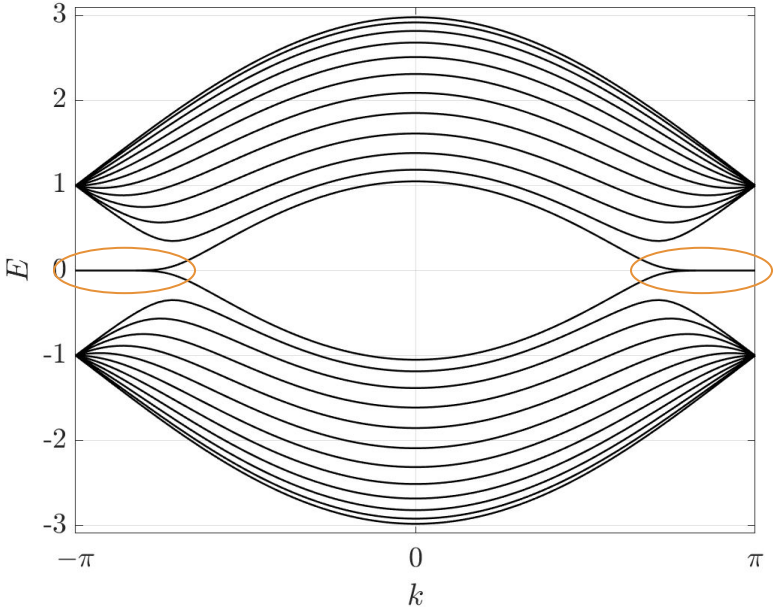
Transfer matrix equation →

Eigenmodes of the system : $\phi_j^{\pm}$

Eigenvalues : $\lambda_j^{\pm}$

Analyses - Dispersion relation





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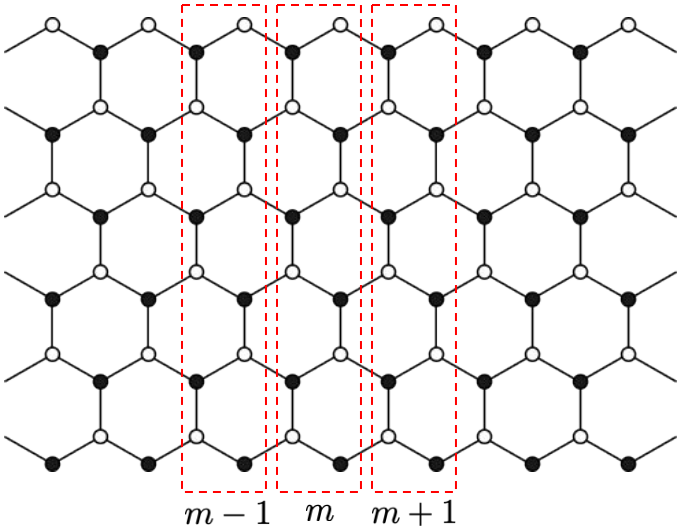
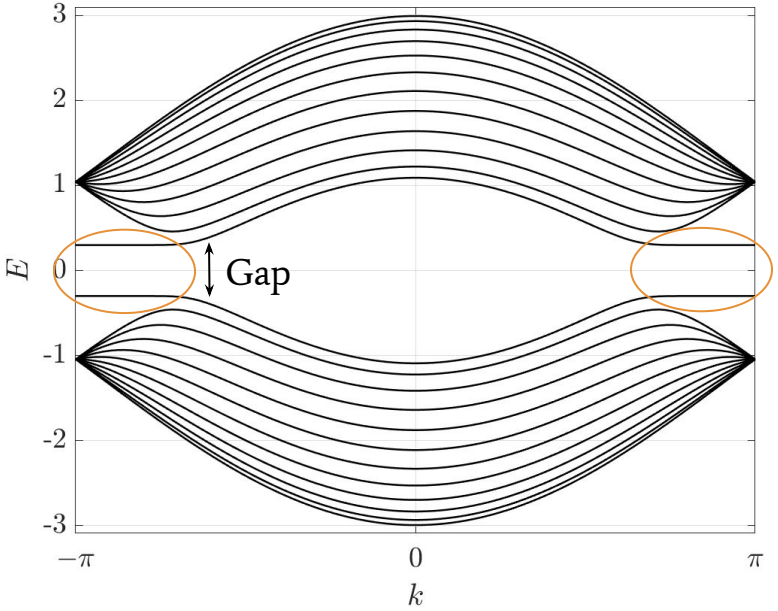
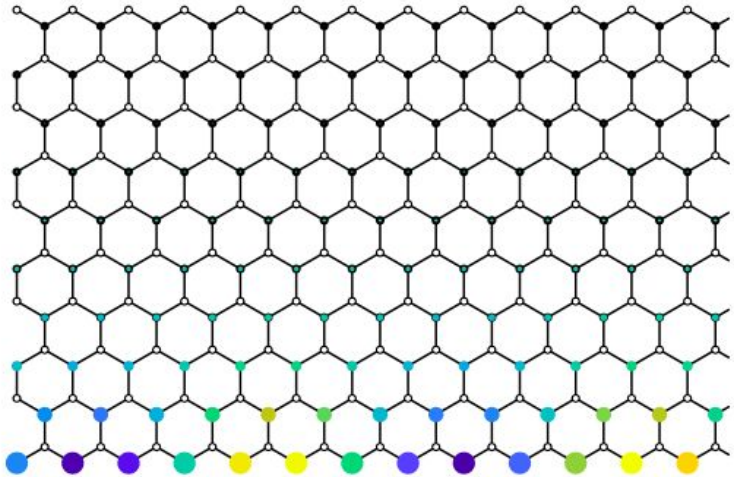
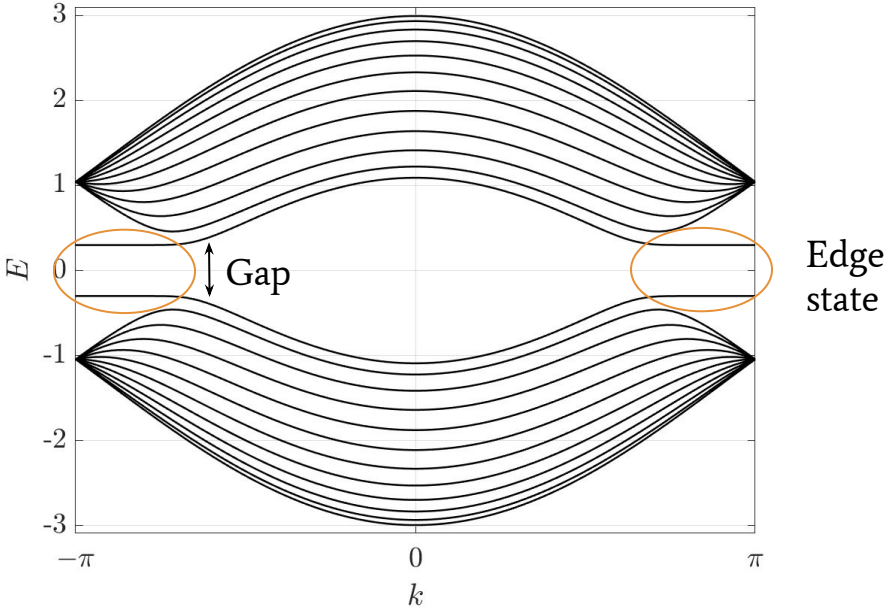




Edge state

Analyses - Dispersion relation





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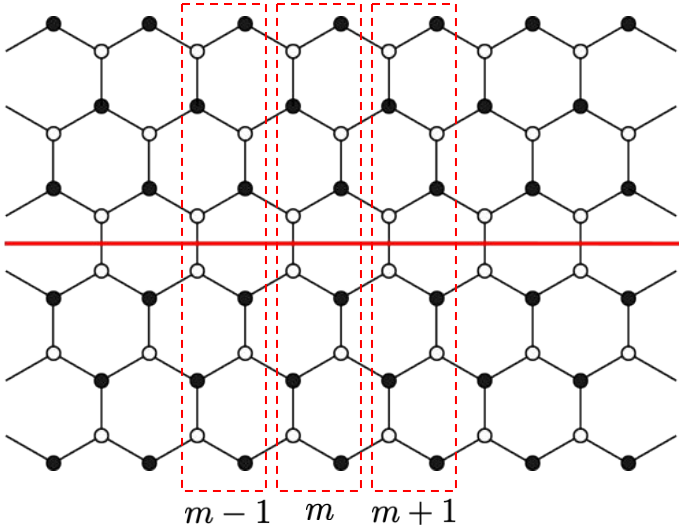

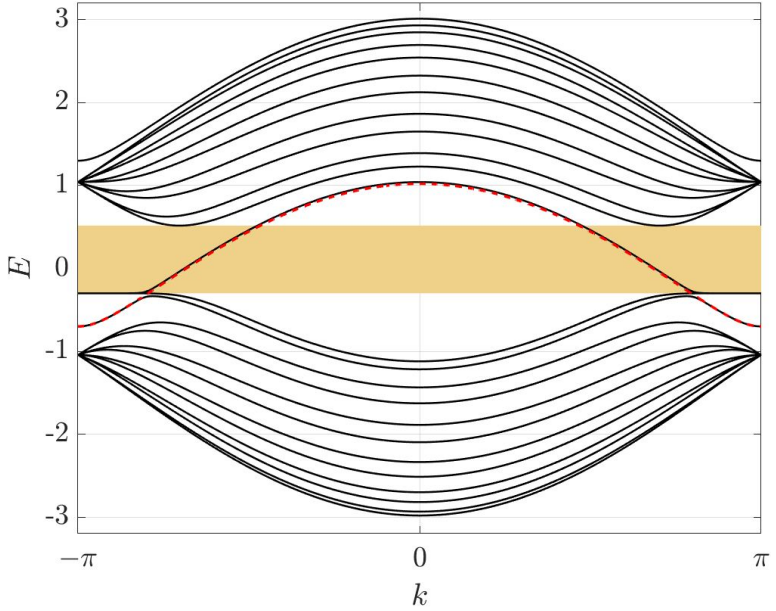

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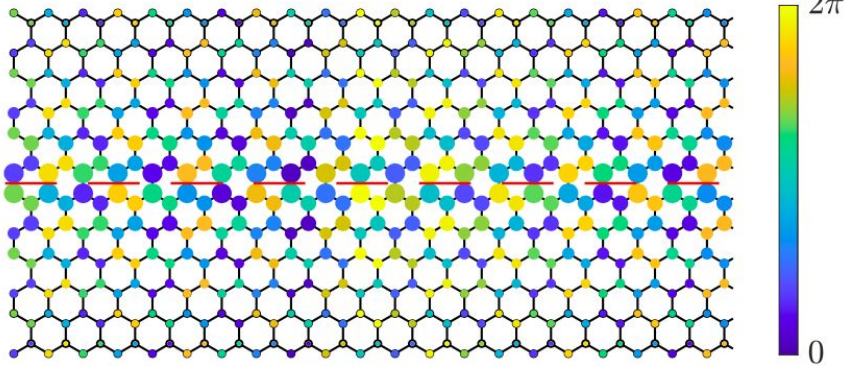
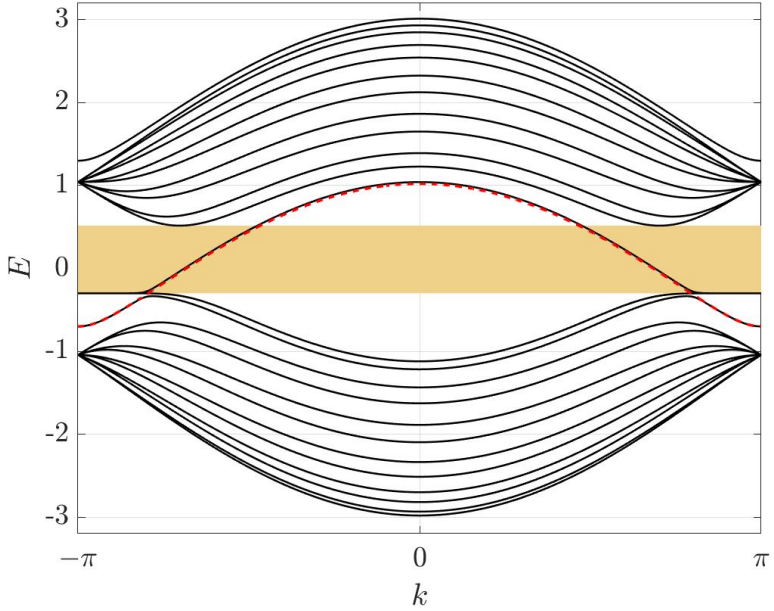
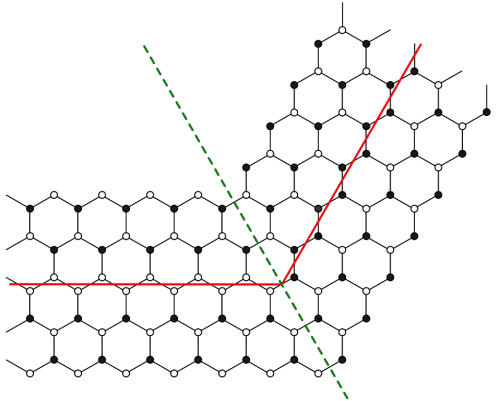


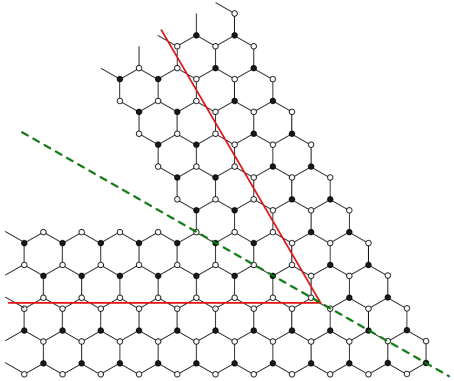


Analyses - Geometrical configurations

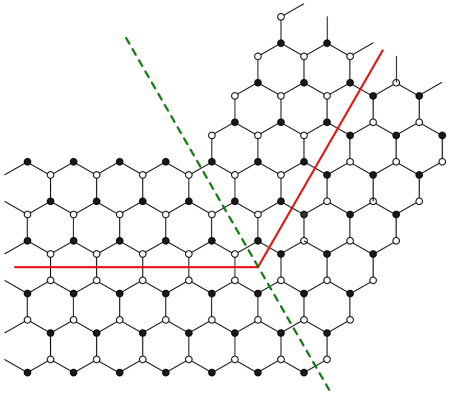
a) Zigzag interface & $\pi/3$ angle



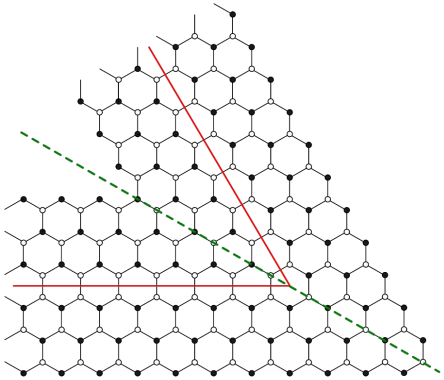
b) Zigzag interface & $2\pi/3$ angle

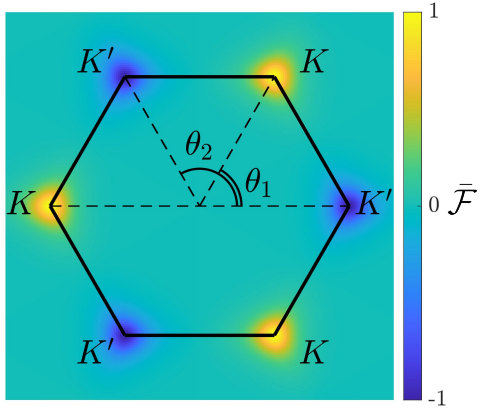


c) Bridge interface & $\pi/3$ angle



d) Bridge interface & $2\pi/3$ angle



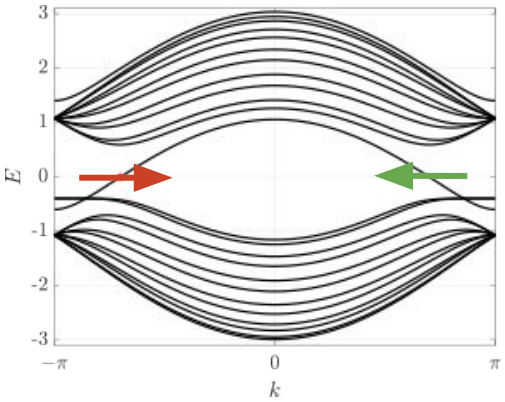


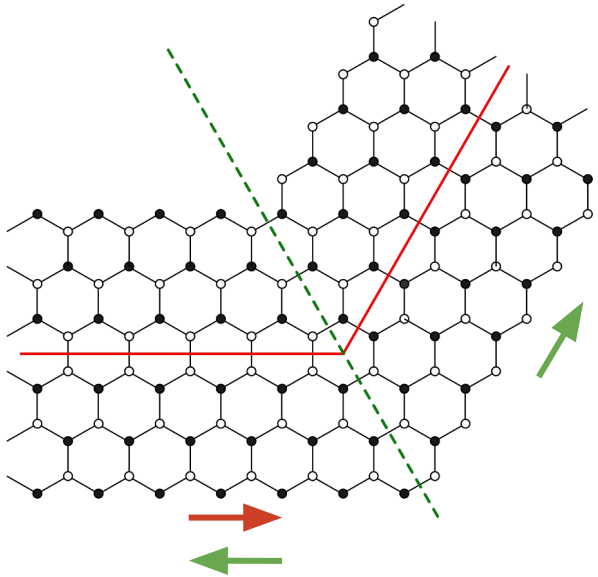
From valley conservation :

- Good transmission from K to K
- No transmission from K to K'.

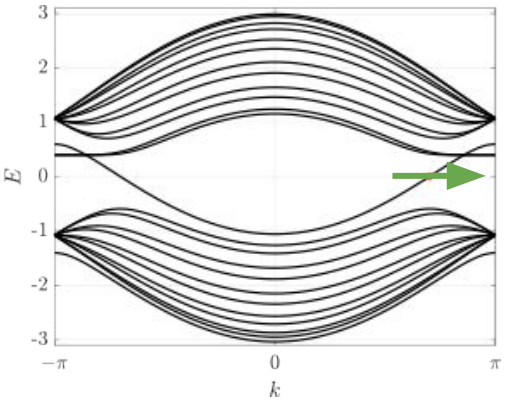
Analyses - Geometrical configurations

Dispersion of the left ribbon



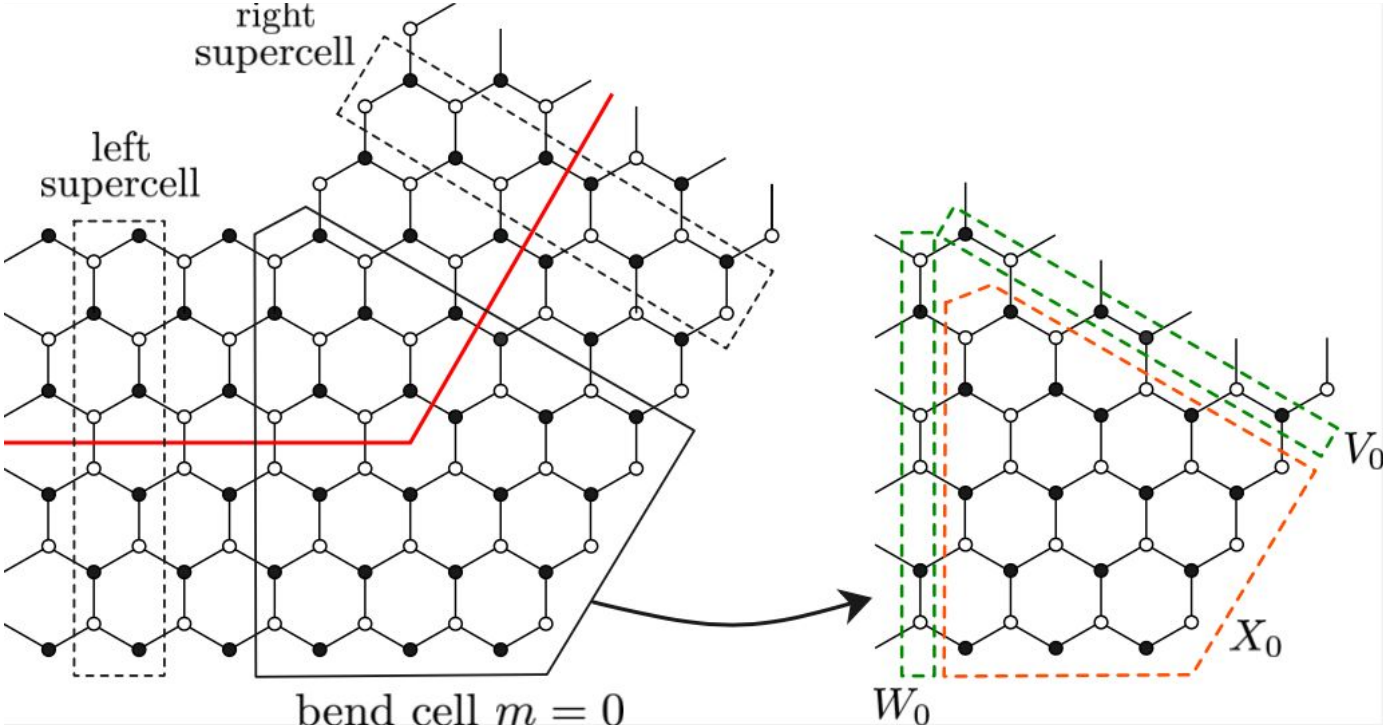


Dispersion of the right ribbon



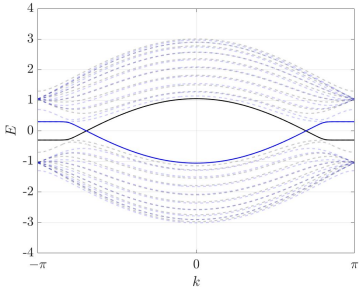
Changing valley is inevitable !

Analyses - Geometrical configurations

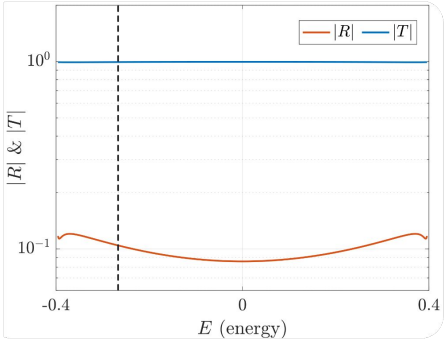


Analyses - scattering vs energy

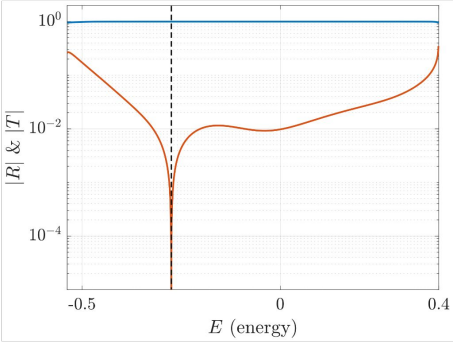
- Symmetric spectrum



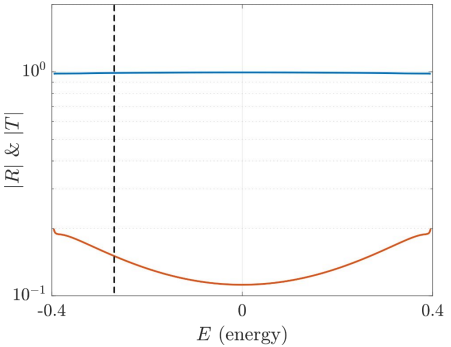
a) Zigzag interface & $\pi/3$ angle



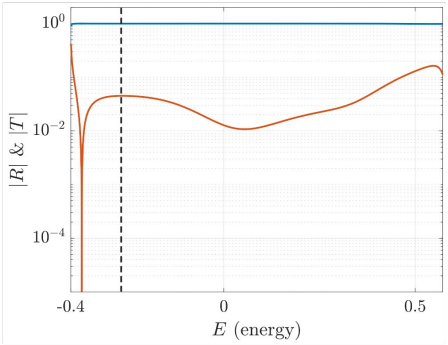
b) Zigzag interface & $2\pi/3$ angle



c) Bridge interface & $\pi/3$ angle



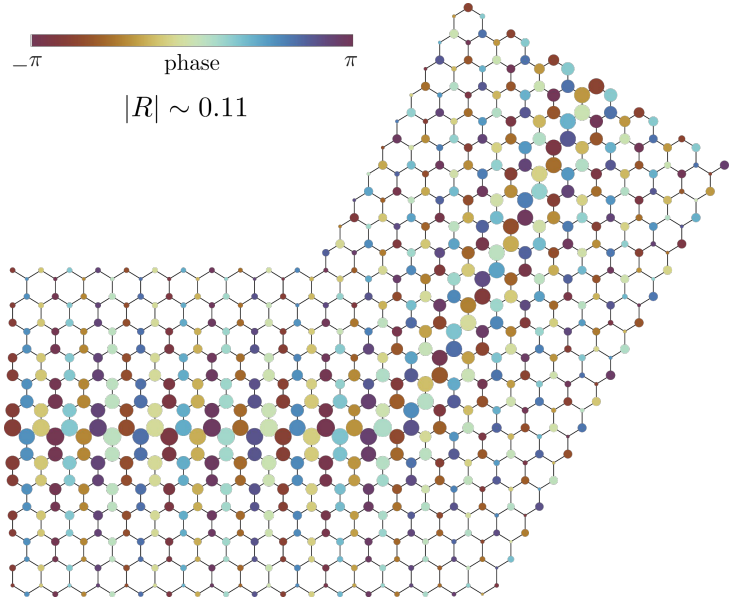
d) Bridge interface & $2\pi/3$ angle

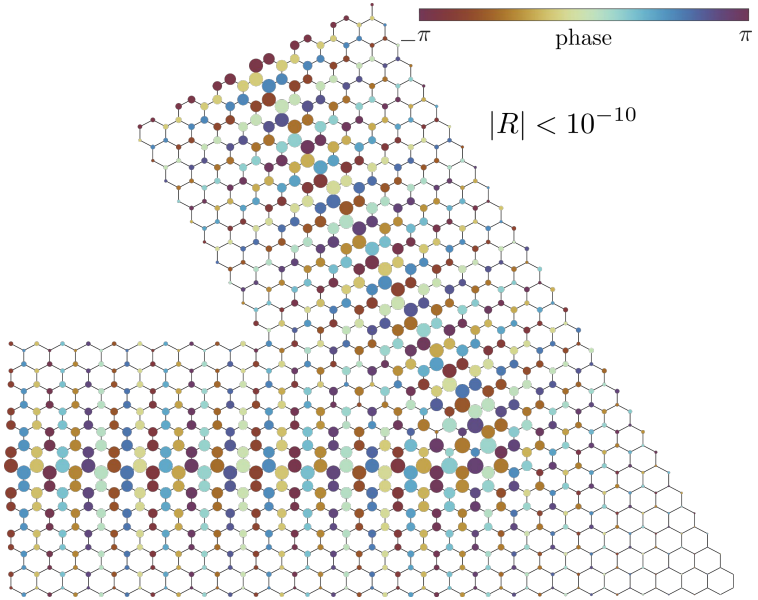


- Presence of zeroes

Modes are well transmitted in all cases !

Analyses - mode reconstruction

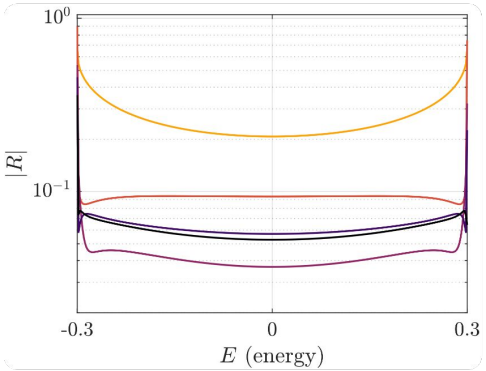




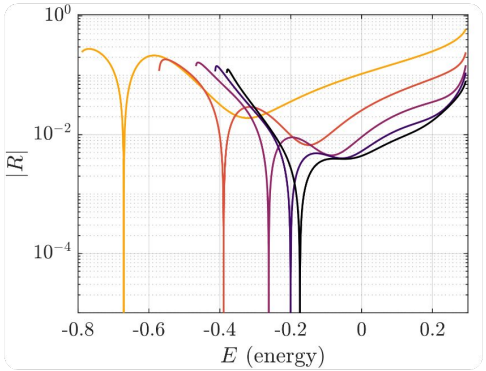
Visually, the modes appear well transmitted but only one has perfect transmission.

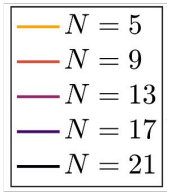
Analyses - Influence of N

a) Zigzag interface & $\pi/3$ angle

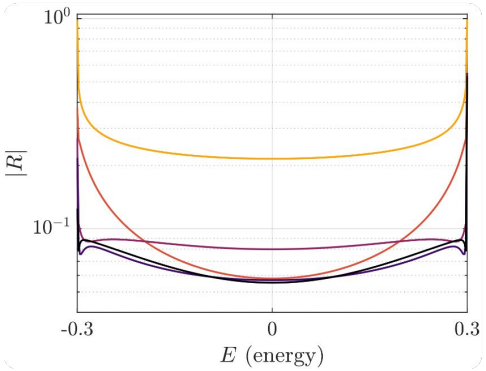


b) Zigzag interface & $2\pi/3$ angle

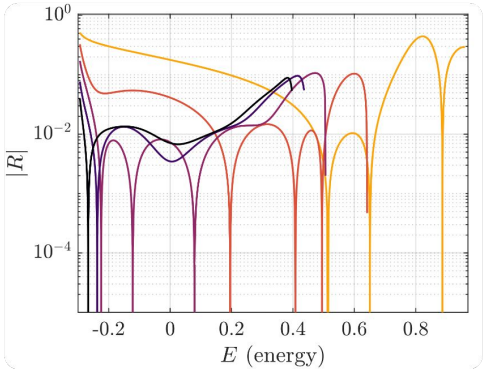


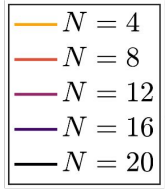


c) Bridge interface & $\pi/3$ angle

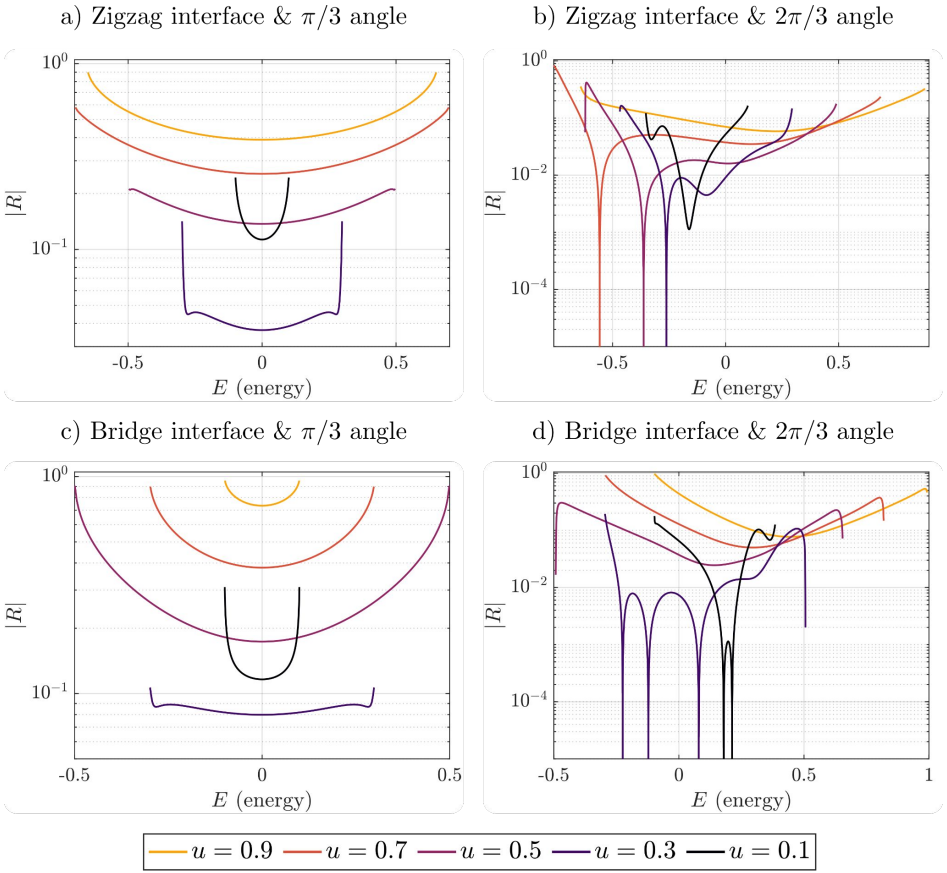


d) Bridge interface & $2\pi/3$ angle



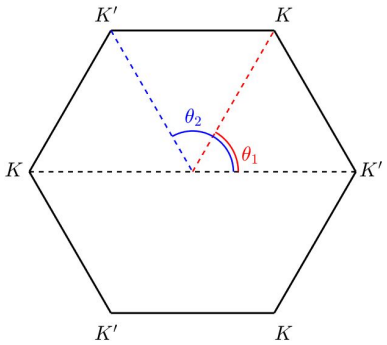


Analyses - Influence of u



Conclusion

- **Valley modes are well transmitted along sharp bends**.
- High transmission is not correlated with the valley index.
- **Non-trivial dependence** on the external parameters.



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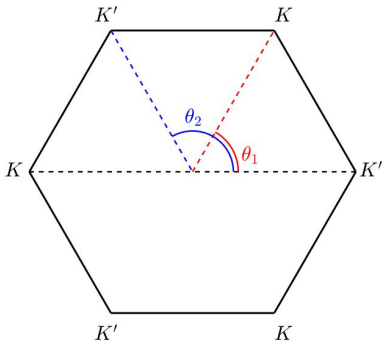
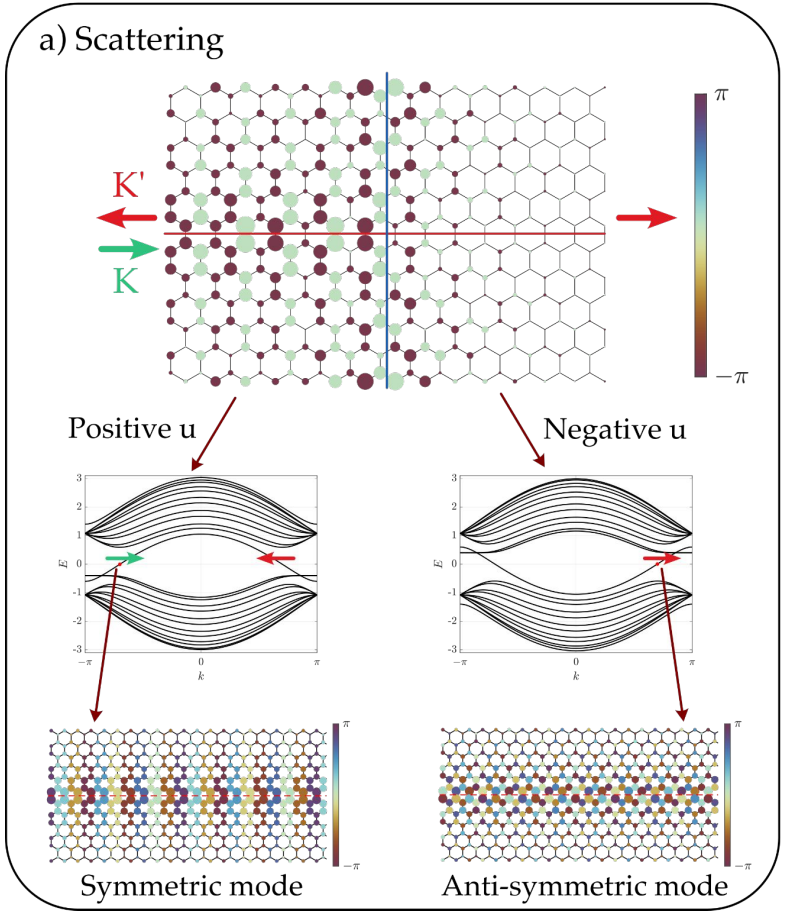
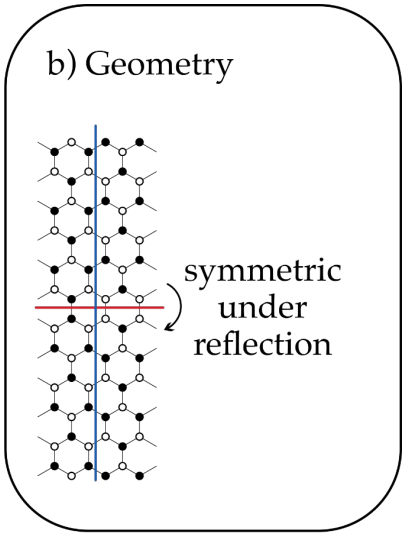


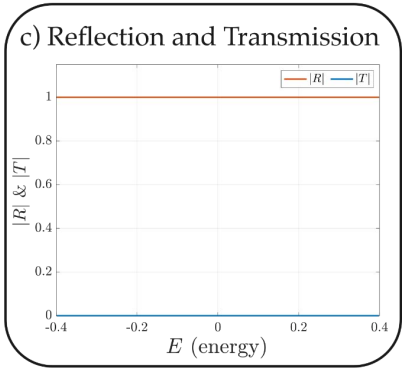
Thank you !

More info in : Torres T., Bellis C., Cottereau R., Coutant A., *arXiv : 2312.04396*

Other situations - 1







Other situations - 2